

A non-local critical problem involving the fractional Laplacian operator

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Along this talk, will be presented some results on the effect of linear, sublinear and superlinear perturbations related with the existence of positive solutions to the following critical elliptic problem involving the fractional Laplacian:

$$\begin{cases} (-\Delta)^{\alpha/2}u = \lambda u^q + u^{\frac{N+\alpha}{N-\alpha}} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases} \quad (1)$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geq 1$, $\lambda > 0$, $0 < q < \frac{N+\alpha}{N-\alpha}$, $0 < \alpha < \min\{N, 2\}$.

For suitable conditions on α depending on q , we prove:

- (i) for $q < 1$, there exists $0 < \Lambda < \infty$ such that Problem (1) have at least two positive solutions for every $0 < \lambda < \Lambda$,
- (ii) for $q = 1$ Problem (1) has at least one positive solution for $0 < \lambda < \lambda_1$,
- (iii) for $q > 1$ the existence of at least one positive solution is shown for every $\lambda > 0$.